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A Simple Treatment of "Snowplow" Models of Explosions

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By using simple bulk conservation laws, some expansion properties of a spherical explosion in an ambient gas are found. These results compare favorably with those resulting from earlier snowplow models which require extensive, detailed calculations.

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1. INTRODUCTION

There are, aside from detailed numerical solutions, several approximate ways of treating the evolution of an explosion in an ambient medium. The appropriate approximation for a given case depends on the nature both of the explosion itself, and on the ambient conditions prior to the explosive energy release. If the explosion occurs in a dense medium, the whole process can be described by continuum mechanics, and a shock will move outward into the initially stagnant ambient medium. Here, "dense" means that the radius of the spherical shock front is very large compared to the ambient mean free path. If the shock is also very strong, the pressure outside the shock front may be neglected in comparison with that inside; this reduces the number of initial parameters entering the problem, and permits solutions of a special type, involving ordinary, rather than partial differential equations. Such solutions have been studied in some detail.[1,2] In the opposite limit, namely an explosion moving into a vacuum, there can be no exterior shock. This problem has been studied using continuum theory for an initially dense interior gas[3] and also for

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1. Taylor, G.I. (1946) The Air wave surrounding an expanding sphere, *Proc. Roy. Soc.* (London) **A186**:273.
2. Sedov, L. I. (1959) *Similarity and Dimensional Methods in Mechanics*, Academic Press, NY.
3. Keller, J. B. (195[illegible]) Spherical, cylindrical and one-dimensional gas flows, *Q. J. App. Math.* **14**:171.

initially tenuous gases having a Maxwellian distribution, using a free molecular flow treatment.[4] These two approaches, interestingly, give quite similar results.

It is for cases in between these two extremes, when the ambient density is neither zero, nor dense enough to allow the use of continuum theory that the models called "snowplows" apply. Between 1964 and 1969 several articles [5,6,7] dealing with explosions occurring in a rarefied atmosphere appeared in the literature. The application of these articles was primarily to high altitude chemical releases and all three utilized a more or less detailed hydrodynamical treatment of the expanding gas produced by the explosion. This involves, in general, the solution of a set of non-linear, partial differential equations. Here, contrariwise, the internal details of the expansion are completely ignored and only the overall conservation of momentum is considered. It is the purpose of this note to illustrate how a simple application of basic principles can, on occasion, be used to obtain results as accurate as extended, much more complicated calculations.

2. PRESSURE FREE EXPANSION

Two of the last three articles cited refer to a "simple snowplow" model. In this model, it is assumed "that the sphere's radius expands so as to conserve kinetic energy, T". Now, the expanding gas that results from the explosion is considered to be a "spherical piston", which as it expands, sweeps up ambient gas in its motion. But sweeping up gas, that is, accretion, is clearly an inelastic process and it cannot be correct to assume that kinetic energy is conserved. What is, to a first approximation, conserved is outward momentum; and the conservation of momentum may, if we assume the gas to expand in a uniform shell, be written down immediately as

$$M_oV_o = MV = (M_o + 4/3\,\pi\,R^3\rho)\,V$$

$$= (M_o + 4/3\,\pi\,R^3\rho)\,\frac{dR}{dt} \qquad (1)$$

Here, t is the time, M_o is the mass of the released gas, V_o the initial outward velocity of the shell, M and V the corresponding quantities at later times; $R = R(t)$ the expansion radius; and ρ the density of the ambient gas. Introducing a non-dimensional variable ϕ given by

$$R = \left(\frac{M_o}{4\pi\rho}\right)^{1/3}\phi \qquad (2)$$

and letting $T_o = E_o$ be the initial kinetic and total energy respectively, assumed equal at $t = 0$[1], we have

4. Molmud, P. (1960) Expansion of a rarefied gas cloud into a vacuum, *Phys. Fluids* **3**:362.
5. Stuart, G.W. (1965) Explosions in rarefied atmospheres, *Phys. Fluids* **8**:603.
6. Holway, L.H., Jr. (1969) Similarity model of an explosion in a rarefied atmosphere, *Phys. Fluids* **12**:2506.
7. Klein, M.M. (1968) Similarity solution for cylindrical gas cloud in rarefied atmosphere, *Phys. Fluids* **11**:964.

$$T = 1/2\, MV^2 = 1/2\, M \frac{M_o^2 V_o^2}{M^2} = 1/2 \frac{M_o^2 V_o^2}{M} \tag{3}$$

$$T/T_o = \frac{M_o}{M} = \frac{M_o}{M_o + 4/3\ \pi R^3 \rho} = \frac{1}{1 + \phi^3/3} \tag{4}$$

In Figure 1, we plot this last, superimposed on Figure 3 of Holway, and see immediately that this simple procedure gives results intermediate between those of Stuart and the later results of Holway.

If, instead of a shell expansion, we assume a similarity solution, $v(r) = r/RV(R)$ where r and v are the radius and corresponding velocity at points interior to the expansion surface, the momentum is given by 3/4 MV; and the kinetic energy by $3/5\ MV^2$. Eqs. (1) and (4) remain valid, however, as does the subsequent integration. It is assumed in this case that the interior density is uniformly distributed.

A natural non-dimensionalization of the time leads to

$$t = \frac{1}{V_o}\left(\frac{M_o}{4\pi\rho}\right)^{1/3} s = \left(\frac{M_o^5}{128\pi^2 \rho^2 E_o^3}\right)^{1/6} s \tag{5}$$

The last form is identical with Holway's if $128 = 423(\gamma - 1)^3$ or $\gamma = 5/3$. Equation (1) in non-dimensional variables is

$$(1 + \phi^3/3) \frac{d\phi}{ds} = 1 \tag{6}$$

or, integrating, with $\phi(0) = 0$,

$$s = \phi + \phi^4/12 \tag{7}$$

In Figure 2, Eq. (7) is superimposed on Holway's Figure 1 and again gives results intermediate between the "simple snowplow" and the Stuart solution on the one hand, and the more recent solution of Holway on the other.

3. EXTERNAL PRESSURE

Although all three of the referenced articles carry their solutions up to and well beyond values of $\phi = 1$, none of them includes the effects of an external pressure. Insofar as these models are applicable to chemical releases, it is clear however that for such radii external pressure effects must play a role in the expansion; and in fact, that for expansion much beyond $\phi = 1$, they will rapidly come to dominate the process. This will occur because the internal momentum is fixed, while the external pressure exerts a force which is proportional to the expanding surface area.

To take account of the pressure in the simplest way, we replace Eq. (1) by

$$\frac{d}{dt}\left[(M_o + 4/3\ \pi\ R^3 \rho) \frac{dR}{dt}\right] = -4\pi\ R^2\, p_o \tag{8}$$

where P_o is the ambient pressure. Now ordinarily $p_o = \bar{c}^2/3\rho$. ($\bar{c}$ is the ambient thermal velocity.) Here, however, material is not reflected, but accretes; hence we set $p_o = \bar{c}^2/6\rho$ to obtain

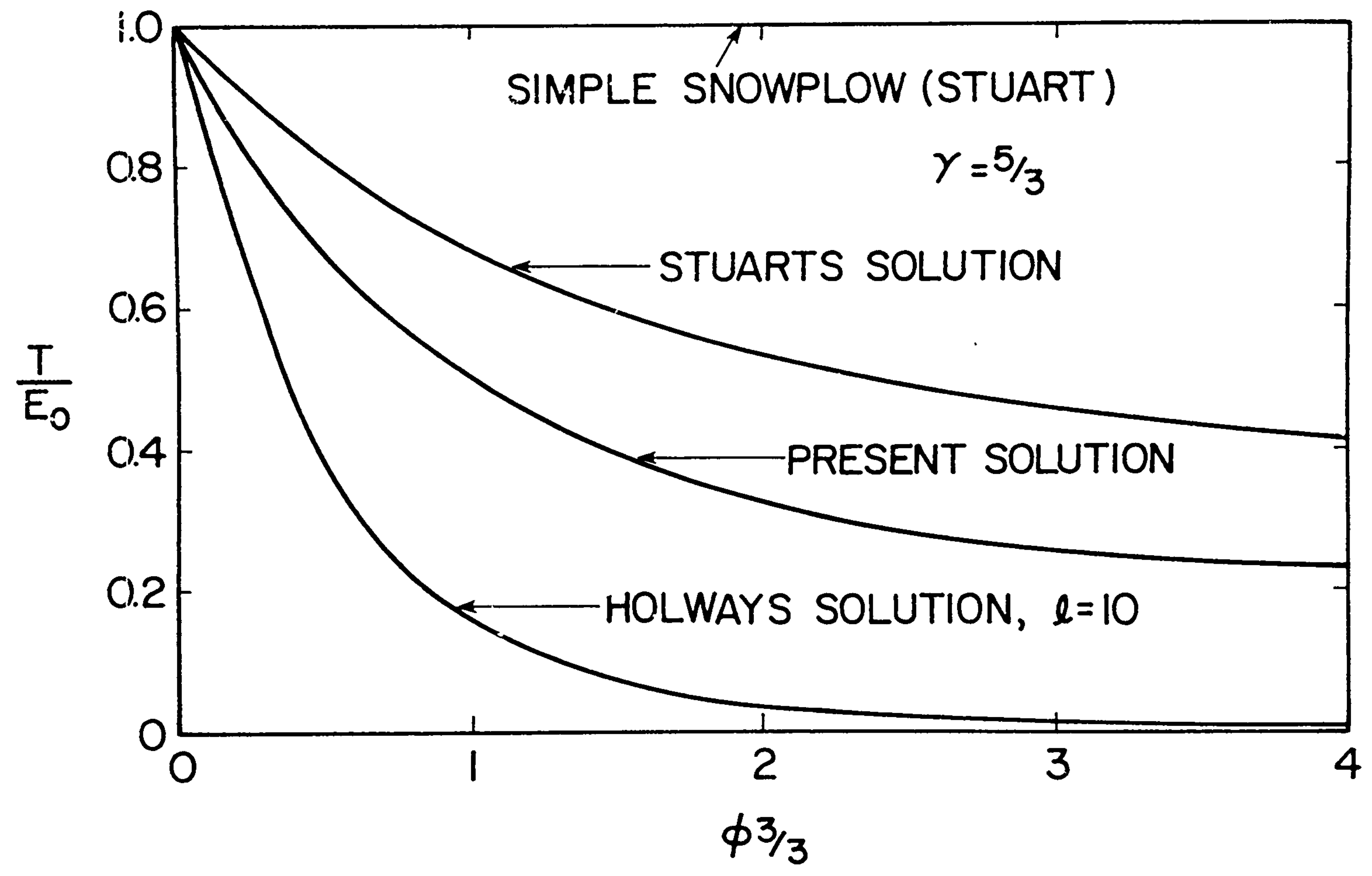

Figure 1. Non-Dimensional Kinetic Energy Versus Non-Dimensionalized Expansion Radius According to Present and Previous Solutions. Here l is related to an (assumed) form of the density distribution (uniform, shell, etc.) and γ is the ratio of specific heats.

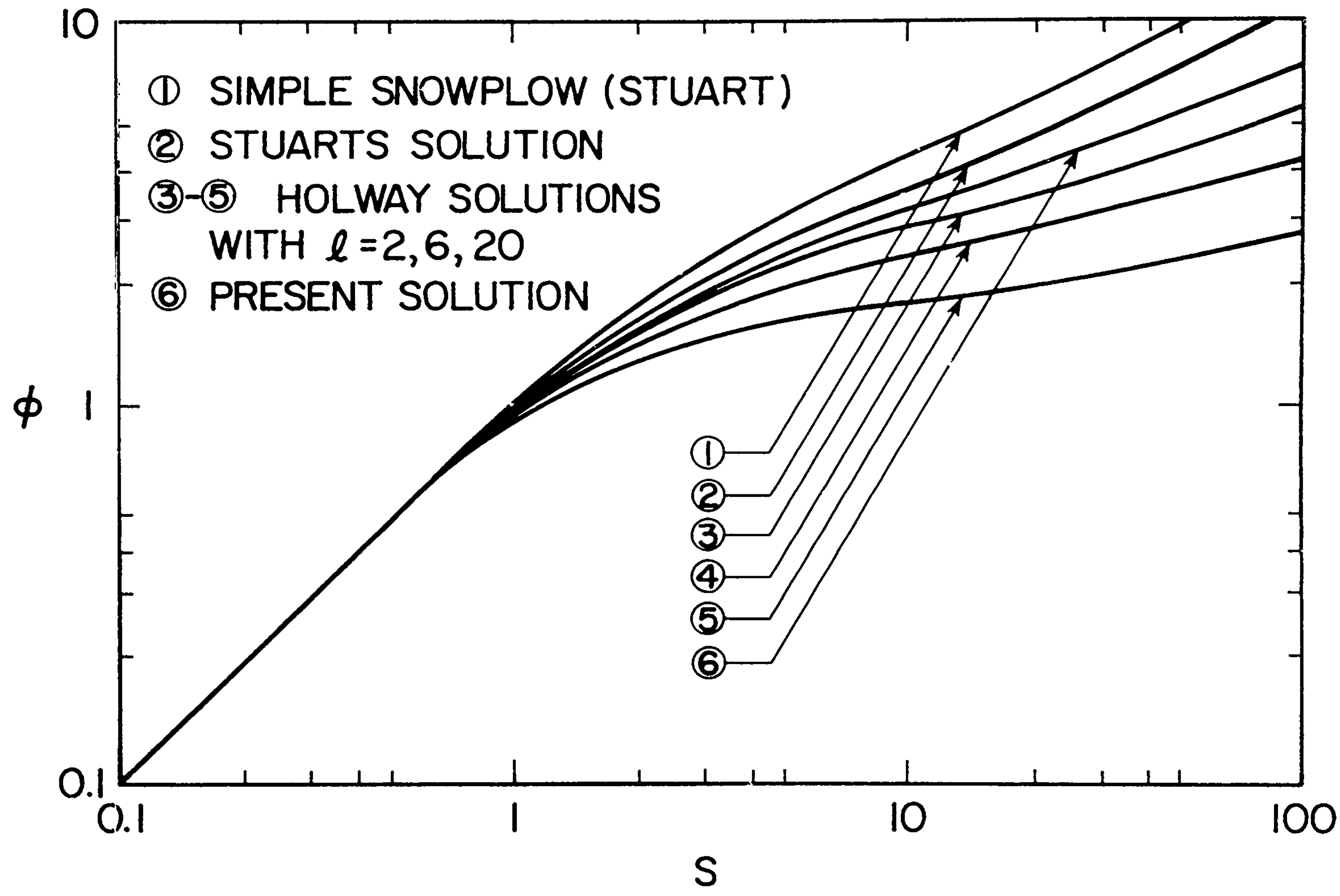

Figure 2. Non-Dimensionalized Expansion Radius Versus Non-Dimensionalized Time, According to Present and Previous Solutions.

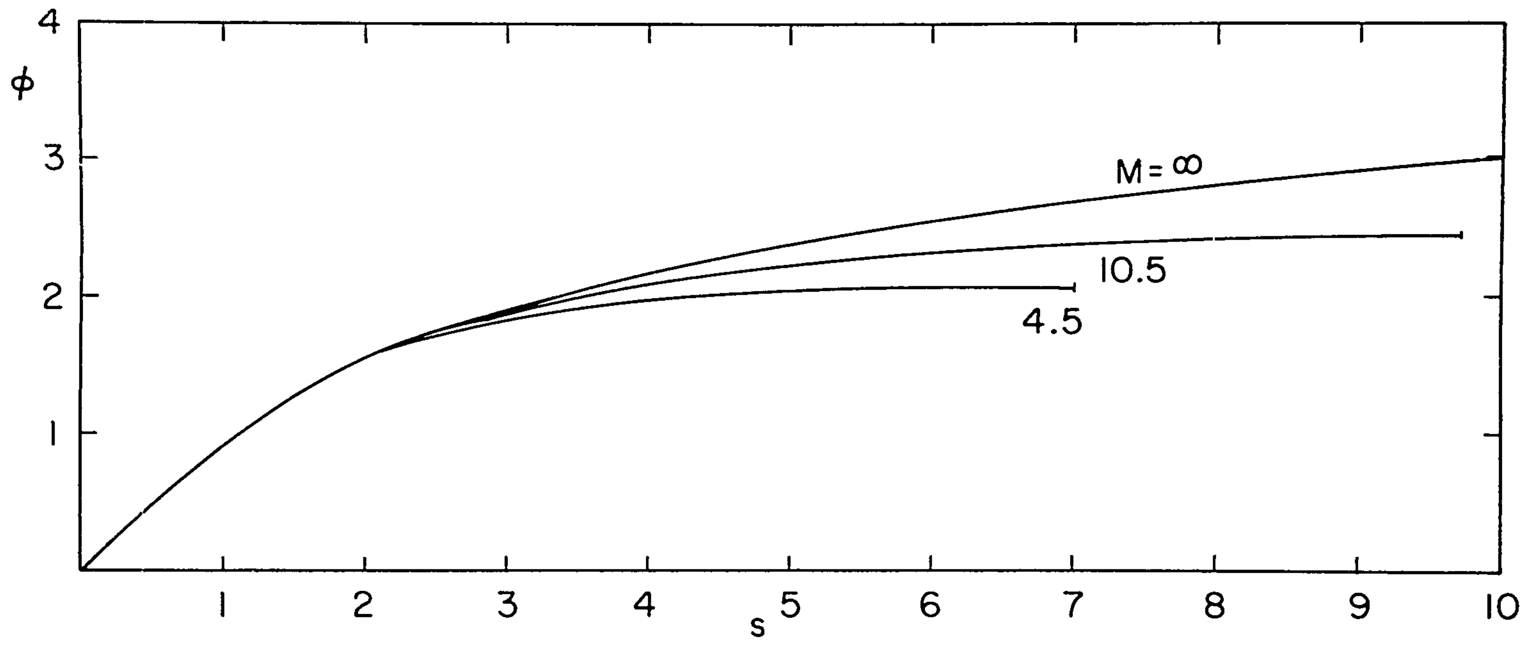

Figure 3. Non-Dimensionalized Expansion Radius Versus Non-Dimensionalized Time, Including Effects of Ambient Pressure. M is the initial expansion Mach number.

$$(M_o + 4/3\ \pi\ R^3\rho)\ \frac{d^2R}{dt^2} + 4\pi R^2\rho\left(\frac{dR}{dt}\right)^2 = -4\pi R^2\rho\ \bar{c}^2/6 \tag{9}$$

Non-dimensionalizing, as before

$$(1 + \phi^3/3)\ \frac{d^2\phi}{ds^2} + \phi^2\left(\frac{d\phi}{ds}\right)^2 = -\phi^2\frac{\bar{c}^2}{6V_o{}^2} \tag{10}$$

Letting a = sonic velocity of the ambient gas, $a^2 = \frac{\gamma}{3}\bar{c}^{\,2}$ so that

$$(1 + \phi^3/3)\ \frac{d^2\phi}{ds^2} + \phi^2\left(\frac{d\phi}{ds}\right)^2 = -\phi^2/2\gamma M^2 \tag{11}$$

where M, the initial ("hybrid") Mach number will not be confused with total mass. A first integral of Eq. (11) is

$$\left(\frac{d\phi}{ds}\right)^2 = \frac{C}{(1 + \phi^3/3)^2} - \frac{1}{2\gamma M^2} \tag{12}$$

When $t = 0$, $\phi = 0$, and $\frac{d\phi}{ds} = \frac{1}{V_o}\frac{dR}{dt}\Big|_o = 1$ so that $C = 1 + \frac{1}{2\gamma M^2}$ and

$$\left(\frac{d\phi}{ds}\right)^2 = \frac{1}{2\gamma M^2}\left[\frac{2\gamma M^2 + 1}{(1 + \phi^3/3)^2} - 1\right] \tag{13}$$

An absolute upper limit on ϕ is now given by $(1 + \phi^3/3)^2 < 2\gamma M^2 + 1$ or $\phi^3(2 + \phi^3/3) < 6\gamma M^2$. If $\gamma = 5/3$ and $M = 10$ (typical values), then $\phi \leq 2.5$; a value smaller than some of those plotted by Stuart and Holway. Because of the ϕ^6 term, this result is rather insensitive to the value of M.

Taking the root of Eq. (13) and separating,

$$\frac{\sqrt{2\gamma}M(1 + \phi^3/3)\ d\phi}{[2\gamma M^2 + 1 - (1 + \phi^3/3)^2]^{1/2}} = ds \tag{14}$$

The left hand side of Eq. (14) leads to an elliptic integral and cannot be integrated in elementary terms. It is most conveniently handled by a numerical integration, and in Figure 3, taking $\gamma = 5/3$, we show the results of the integration for several values of the initial Mach number, M. Since $M = \infty$ is equivalent to the pressure free case, the inclusion of the external pressure brings the values of ϕ closer to those obtained by Holway; although it must be noted that the validity of snowplow solutions for values of $\phi > 1$ is questionable.

In spite of the extreme simplicity of these considerations, the results obtained, both with and without external pressure, compare favorably with those of the earlier, more detailed calculations.

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1. Taylor, G.I. (1946) The Air wave surrounding an expanding sphere, *Proc. Roy. Soc.* (London) **A186**:273.
2. Sedov, L. I. (1959) *Similarity and Dimensional Methods in Mechanics*, Academic Press, NY.
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